

higher, transcendental functions of Abel.

The deeper significance of this discovery can only be hinted at in this installment, and will be taken up in more depth later, but it can be illustrated by the animation illustrated in Figure 11, which expresses the principle of least-action with respect to an elliptical function. Riemann demonstrated that all elliptical functions, being functions formed by the interaction of two connected principles, are expressed in the complex domain as surfaces with two boundaries (these boundaries are marked in green) [SEE inside front cover]. Each boundary changes differently, but connectedly, with the other, causing corresponding changes in the minimal pathways, while at all times maintaining the overall harmonic relationship of the function. In other words, the characteristic curvature of these least-action pathways is determined, in this case, by the connected interaction of two distinct principles.

A comparison of this to the previous examples indicates what Riemann emphasized: That the only way to fundamentally change the characteristic of action of a physical process, is by the addition of the action of a new principle. This more advanced question will be investigated more thoroughly in future Pedagogicals.

A suggestive example from econom-

ics can help illustrate this principle. What is the relationship between all physical-economic relationships, and the economic boundary conditions of physical infrastructure and cultural development? What is the relationship between these boundary conditions, and the singularities represented by the introduction of new technologies? What is the effect on all economic relationships, of a change, positive or negative, in these physical-economic boundary conditions?

Four years after Riemann's death, Karl Weierstrass criticized Riemann's application of "Dirichlet's Principle" on formal mathematical grounds. Weierstrass contended that it was inappropriate to speak mathematically of least-action, unless a formal mathematical proof could be presented proving that a mathematical minimum, or maximum, existed. While it is possible to produce a formal mathematical example which has no minimum, all *physical* processes are characterized by bounded least-action. For example, as Nicolaus of Cusa showed, there is no absolute maximum or absolute minimum polygon, because the polygon is bounded maximally by a circle (which is not a polygon) and minimally by a line (which is also not a polygon). Or, while a mathematical catenary can be extended

into infinity, the physical catenary is always bounded by the hanging points. For Riemann, as for Gauss and Dirichlet, Weierstrass's demand for a formal mathematical proof of a minimum, was less than unnecessary: It was a sophistry. The universal physical principle of least-action was sufficient to supply the proof.

Weierstrass's critique was seized upon by the formalists, who were desperate to roll back the achievements of Kästner, Gauss, Dirichlet, Jacobi, Abel, Riemann, *et al.*, and return science to the slavish days of Euler, Lagrange, and d'Alembert. Consequently, while the form of Riemann's discoveries has been widely discussed, the substance of his thinking has by and large been suppressed, until it found new life in the more advanced discoveries of Lyndon LaRouche.

—Bruce Director

1. See, e.g., Bruce Director, "The Long Life of the Catenary: From Brunelleschi to LaRouche," *Fidelio*, Spring 2003 (Vol. XII, No. 1).
2. See G.W. Leibniz, "Two Papers on the Catenary Curve and Logarithmic Curve (*Acta Eruditorum*, 1691)," trans. by Pierre Beaudry, *Fidelio*, Spring 2001 (Vol. X, No. 1).
3. See Bruce Director, *Riemann for Anti-Dummies*, Part 53: "Look to the Potential," Dec. 21, 2003 (unpublished).

Part 2

Lejeune Dirichlet and the Mendelssohn Youth Movement

When Lejeune Dirichlet, at 23 years of age, worked with Alexander von Humboldt to make microscopic measurements of the motions of a suspended bar magnet in a specially-built hut in Abraham Mendelssohn's garden, he could hear, in the nearby summer house, the Mendelssohn youth movement work through the voicing of J.S. Bach's *St. Matthew Passion*. Felix and Fanny Mendelssohn, brother and sister aged 19 and 23, respectively, were the

leaders of a group of 16 friends who would meet every Saturday night in 1828 to explore this "dead" work, unperformed since its debut a century earlier by Bach.¹

The two simultaneous projects in the Mendelssohn garden at Berlin's 3 Leipziger Strasse are a beautiful example of Plato's Classical education necessary for the leaders of a republic: The astronomer's eyes and the musician's ears worked in counterpoint, for the

higher purpose of uniquely posing to the human mind, *how the mind itself worked*. As described in the *Republic*, Book 7, the paradoxes of each "field"—paradoxes (such as the "diabolus") that, considered separately, tied up in knots the "professionals" of each—taken together would triangulate, as it were, for the future statesman, the type of problems uniquely designed to properly exercise the human mind. After all, such a mind would have to master more than astron-

omy and music, simply to bring before it a series of paradoxes, so as to be made capable of dealing with the much more complicated affairs of a human society. To oversimplify: Since the mind does not come equipped with a training manual, the Composer of the universe created the harmonies of the heavens and of music, as, for example, a mobile above a baby's crib.

In that hut, Dirichlet would be taking measurements as part of making a geomagnetic map of the Earth. The audacity in thinking that these miniscule motions of the suspended bar magnet could capture such unseen properties, posed certain appropriate questions to Dirichlet. (Gauss's geodetic surveying a decade earlier was paradigmatic of the sort of project that mined such riches out of the ostensibly simple affair of determining where one actually was! But this also applies to locating oneself in the process of a proper daily political-intelligence briefing.) Similarly, the 16 youths working to solve amongst themselves the complicated inter-relationships of Bach's setting of the Passion story as related by St. Matthew, would have been forced to grapple with the scientific problem of ascertaining what our Maker would have in store for us, in their attempt to map their own souls. (Just for starters in their "performance" questions: How does Jesus intone what he says? How does the chorus/audience respond to Jesus, and sometimes to each other? etc.) The following historical sketch is offered as a few measurements, but instead of using a suspended magnetic bar, we will use a few years of Dirichlet's life, and thereby try to triangulate some of the important characteristics for a map of the culture that created the world which we are challenged to master today.

Humboldts and Mendelssohns

Dirichlet's patron, Alexander von Humboldt, along with his brother Wilhelm, had studied in the 1780's with a host of pro-American Revolution leaders in Europe, notably including the Mendelssohns' famous grandfather, Moses. (Those studies can be investigated by reading Moses Mendelssohn's Leibnizian work, *Morgenstunden*, or



Fanny Mendelssohn

Morning-Studies, which describe the lessons that he gave to his son Joseph, and to the young Humboldt brothers.) Later, two of Moses's sons, Joseph and Abraham, ran the Mendelssohn Bank, which financed many of Alexander von Humboldt's scientific expeditions and projects. Abraham Mendelssohn, the father of Fanny, Felix, Rebecca, and Paul, had constructed, in his garden at 3 Leipziger Strasse, a special magnetically neutral observation hut for Humboldt to measure minute magnetic fluctuations. Humboldt brought Dirichlet to Berlin in 1828, where he was one of a five- or six-man team that shared observational duties with Humboldt, in their mapping of the actual geomagnetic shape and potential of the Earth.

In 1827-28, Humboldt gave public lectures at the Singakademie Hall on physical geography—unusually, open to both men and women. Fanny Mendelssohn described in a letter to her friend Klingemann: "[T]he course is infinitely interesting. Gentlemen may laugh at us as much as they will; it is wonderful in this day and age for us to have an opportunity to hear something sensible, for once. I must further inform you that we are attending a second lecture series, given by a foreigner on experimental physics. This course, too, is being attended mainly by women."²

Humboldt's public lectures were an extension of his instruction at Berlin's famous Friedrich Wilhelm University,



Felix Mendelssohn

which had been established in the previous decade by his brother Wilhelm. While Felix Mendelssohn attended the University that year, a collaborator of Humboldt at the University, Philip August Boeckh, the great philologist, was living as a tenant in the Mendelssohn home. (Years later, Felix would compose music for the staging of Boeckh's German translation of Sophocles' play, *Antigone*.) Humboldt also organized the Berlin scientific congress of August 1828—a conference that Metternich would find most dangerous. For the several weeks that Gauss stayed at Humboldt's home for the conference, they could discuss the implications of the geodetic and geomagnetic projects. Finally, the representative from England, Charles Babbage, the noted promoter of Leibniz's analytic methods, against those of Newton and the Newtonians, expressed his delighted amazement at the culturally optimistic Mendelssohn household. It was in such circumstances that Dirichlet entered into the Mendelssohn youth movement.

The Mendelssohn Youth Movement

Fanny reports on the scene in a Dec. 27, 1828 letter to Klingemann: "Christmas-eve was most animated and pleasant. You know that in our house there must always be a sort of 'jeune garde' ['young guard'], and the presence of my brothers and the constant flow of young life exercise an ever attractive influence. I must mention

Dirichlet, professor of mathematics, a very handsome and amiable man, as full of fun and spirits as a student, and very learned." Fanny's sister, and Dirichlet's future wife, Rebecca, was also at that Christmas party. We may assume that some or all of the 16-member "Saturday-night chorus" were there, too.

Fanny's long-time love, Wilhelm Hensel, back in Berlin for two months now, was there. He had just returned from five years of study of Renaissance art in Italy. Wilhelm, now 33, and a talented artist, had fought as a young man in the German Liberation Wars against Napoleon. Now, he had returned to Berlin to win Fanny as his wife (which involved conquering Fanny's mother, Leah). A month later, the engagement was announced.

Fanny also mentions three of the suitors of Rebecca (who would all lose out to Dirichlet):

- Professor Eduard Gans: "We see him very often, and he has a great friendship for Rebecca, upon whom he has even forced a Greek lesson, in which these two learned persons read Plato. It stands to reason that gossip will translate this Platonic union into a real one . . ." Gans had been active in Jewish causes early on, but he converted in 1825, so that he could become a professor.³

- Johann Gustav Droysen, historian and philologist: Although he was only 19 years old, Fanny recognized in him

"a pure, poetic spirit and a healthy amiable mind." Droysen published a translation of Aeschylus, and a famous work on Alexander the Great, both before he was 25.

- Heinrich Heine, poet: "Heine is here. . . . [H]is *Reisebilder* contain[s] delightful things; and though for ten times you may be inclined to despise him, the eleventh time you cannot help confessing that he is a poet, a true poet!" Once, he sent, via his close friend Droysen, his greetings to the 18-year-old Rebecca: "As for chubby Rebecka, yes, please greet her for me too, the dear child she is, so charming and kind, and every pound of her an angel." It seems that Heinrich Heine's brand of courtship of Rebecca was no different from his treatment of everything else in life.

The 'St. Matthew Passion'

Now picture Dirichlet in the observation hut in the garden at 3 Leipziger Strasse. Close by is the summer house, where Felix and Fanny worked out, with four hands at the piano, the voicing and composition of Bach's *St. Matthew Passion*—not performed since Bach premiered it in 1729. In January 1829, soon after Dirichlet had arrived on the scene of the Mendelssohn youth movement, Eduard Devrient and Felix Mendelssohn decided upon an historic March public performance, despite the discouragement of the musical authori-

ties. As described years later by Fanny's son, the appropriately named Sebastian Hensel: "Only just then the most intelligent musical people began to comprehend that something must be done to bring this treasure to daylight, and that this was from a musical point of view the greatest task of the period."

After hiring a hall, with a performance only six weeks away, the chorus swelled from 16 to 400, and the initial group had the "Monge brigade" project of rapidly educating all the newcomers. Fanny described this rare and sublime process: "People were speechless with admiration, and faces grew long with astonishment at the idea that such a work could have existed unbeknownst to them. . . . Once they grasped that fact, they began studying the work with warm and veritable interest. The enthusiasm of the singers, from the first rehearsal on; how they poured their heart and soul into the work; how everyone's love of this music and pleasure in performing it grew with each rehearsal . . . [all this] kept renewing the general wonder and astonishment." This process created "so lively and detailed an interest that all the tickets were sold the day after the announcement of the concert, and they had to refuse entrance to more than a thousand people. . . . [At the concert itself,] I was sitting in the corner [of the massive chorus] so as to see Felix well, and I had arranged the strongest alto voices near me. The choruses were impassioned with extraordinary strength tempered with a touching tenderness, as I had never heard them before. . . . [A] peculiar spirit and general higher interest pervaded the concert, that everybody did his duty to the utmost of his powers, and many did more . . ."

And, after the sublime, the ridiculous: At least one Berliner seemed to remain untouched. After the concert, at a celebratory dinner, Devrient's wife, Therese, sat between Felix and an obnoxious professor, who kept trying to get her drunk: "He clutched my wide lace sleeve in an unrelenting grip . . . to protect it, he said! And would every so often turn toward me; in short, he so



Rebecca Mendelssohn,
drawing by Wilhelm Hensel.

The Granger Collection



Heinrich Heine, 1826/28.

The Granger Collection

plagued me with his gallantries that I leaned over to Felix and asked: 'Tell me, who is this idiot beside me?' Felix held his handkerchief over his mouth for a moment—then he whispered: 'The idiot beside you is the celebrated philosopher Hegel!' ”⁴

Such were the circumstances of Dirichlet's first year in Berlin. Dirichlet and Rebecca Mendelssohn were engaged in 1831, and married in 1832. In Mendelssohn family discussions and debates, they were taken as the most revolutionary of the group. The couple had four children. Rebecca died late in 1858, age 47—evidently of a type of stroke similar to what had felled her older sister Fanny at 43, and brother Felix at 39, a decade earlier. Dirichlet's compromised health declined further, and he followed her to the grave five months later, on May 5, 1859.

A Parallel Story from Paris

When he was 17, Dirichlet was sent to study in Paris, at which time he was studying Gauss's *Disquisitiones Arithmeticae*. According to Sebastian Hensel, Dirichlet was introduced there to General Foy by a republican associate of Dirichlet's parents, one Larchet de Charmont.⁵ Foy employed Dirichlet as a tutor in his household from the summer of 1823 until Foy's death in November 1825. Foy was in France's chamber of deputies, and was the leader of the opposition to the royalist restoration wrought by the 1815 Congress of Vienna. Dirichlet thrived in this environment: "[I]t was very important for his whole life that General Foy's house—frequented by the first notabilities in art and science as well as by the most illustrious members of the chambers—gave him an opportunity of looking on life in a larger field, and of hearing the great political questions discussed that led to the July Revolution of 1830, and created in him such a vivid interest."⁶

The July Revolution of 1830 was led by Lafayette, and was at best a mixed affair. It overthrew the reactionary arrangements of the Congress of Vienna, and set up a tenuous arrangement whereby Louis Philippe, the "Citizen

King," would be a constitutional monarch. Lafayette gambled that this might work, as the "Citizen King" had pledged to be subservient to the written constitution. Two items of note reflect Foy's connections to the 1830 Revolution: In October 1825, a few weeks before his death, Foy had troubled himself to write to Lafayette; and in 1823, Foy had sent from his care Alexandre Dumas to be Foy's agent in the household of Louis Philippe. (The future author was then 21, Dirichlet's senior by three years.) Later, in 1830, Dumas would serve as a captain in Lafayette's National Guard.

Dumas had sought Foy's guidance, as Foy himself had earlier, in the 1790's, looked to Dumas' father, General Alexander Davy Dumas, as his military and political leader. General Dumas was a hero of the French army, who became an early opponent of Napoleon's imperial ambitions. He was part of the 1798 invasion of Egypt, but was imprisoned by Napoleon from 1799 to 1801 for publicly opposing Bonaparte's imperial turn. (Similarly, Beethoven at this time had hopes for Napoleon that he quickly recognized were greatly mistaken.) Afterwards, Napoleon's harsh treatment of General Dumas led to his early death in 1806, at age 44 (when his son was only 4 years old).

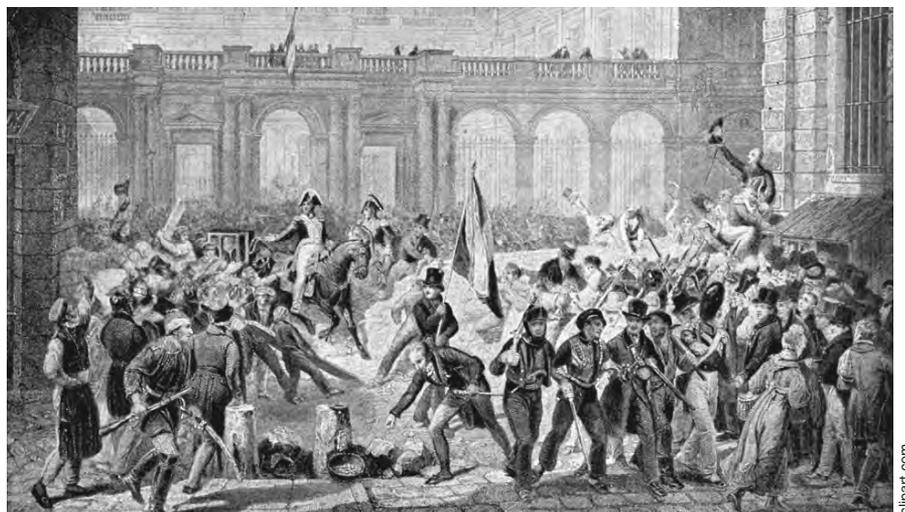
After Foy died in November 1825, there was a competition between

Alexander Humboldt and Joseph Fourier for Dirichlet's services. Fourier, according to Hensel, "tried to avail himself of Larchet de Charmont's influence, to induce him [Dirichlet] to return to Paris, where he felt sure it was his vocation to occupy a high position at the Academy."⁷ Humboldt arranged for Dirichlet, then 21, to teach at Breslau, 1826-28, and then brought him to Berlin in 1828, where he was the professor of Mathematics at the Berlin Military Academy, and where he joined the Mendelssohn youth movement.

Lafayette, Dumas, Galois, Poe, Heine

Alexander von Humboldt returned to Paris in 1830 because of the ripened political situation. Augustin-Louis Cauchy—the Emperor of mathematics—had to flee Paris in July 1830, when his King was deposed. For a short period, Lafayette thought that they could control the new "Citizen King," Louis Philippe. However, within a few months, the financiers moved in to gain the upper hand in running the King. In December 1830, they succeeded in arresting the 19 leaders of Lafayette's republican National Guard, the key defenders of the constitution. Lafayette testified at the March 1831 trial, and the jury found them all not guilty.

At the celebratory dinner for



Paris, the July Revolution of 1830. Lafayette's attempt to establish a constitutional monarchy under Louis Philippe (shown on horseback) proved a failure.

the released “19” were, among others, Lafayette, Dumas, and another brilliant student of Gauss’s work, Evariste Galois. (The latter had been, along with Neils Abel, a victim of Cauchy’s ham-handed skulduggery as head of the French Academy of Science.) At the dinner, Galois evidently made a notorious toast to Louis Philippe’s health, while



Mathematician Evariste Galois

putting his other hand on his sword, and adding that the King had better not fail in his duty to the constitution. Dumas reports that, at that point, several of the attendees, including himself, jumped from the windows of the hall, fearing, accurately, that the spies at the event would bring the police.⁸ Galois was arrested, tried, but, when the jury refused to convict him, released.

He was re-arrested that summer, 1831, by the police prefect, Gisquet, for wearing a republican guard uniform in public. Gisquet avoided the pathway of the unsuccessful trials, and instead kept him in jail until the next spring—when his release, and the set-up of a fatal “duel,” fell hard one upon the other. When Galois’ suspicious death roused a crowd to come to his funeral, and a public accounting was threatened, Gisquet carried out, the night before the funeral, pre-emptive arrests of Galois’ friends.

Which of these events in Galois’ last year, 1831-32, were attended by Edgar Allan Poe, then visiting Paris, is unclear,

but clearly Poe’s “The Purloined Letter” skewers Gisquet (the “prefect G—”), and, by inference, celebrates the “poet-mathematician” Galois. While Poe *does* explicitly refer to the mathematician Charles Auguste Dupin (the historical figure who, literally, was a member of the Monge brigade, having been taught directly by Monge),



Edgar Allan Poe

Poe’s “poet-mathematician” image does not need to be reduced to one individual. However, the politically sensitive case of Galois at the time of Poe’s presence in Paris, and the reference to the “prefect G—,” make it clear that the Galois case would have been understood by astute readers of Poe’s time. Regardless, Poe’s “poet-mathematician” image

would appropriately apply to any of the leading (1820’s) students of Gauss: Galois, Abel, or Dirichlet. So, once again, as in the garden of 3 Leipziger Strasse, we find that unity of the arts and physical sciences characteristic of the republican geniuses of the day.

Finally, Heine, upon the news of the July Revolution, decided to leave Berlin for Paris. He would have been there, with Alexander von Humboldt, during these events. His early work in Paris during this period is reflected in his *The Romantic School*, where he diagnosed for the French and the Germans, the evil medievalism of the cultural string-pullers who had deliberately set out to murder the Germany of Moses Mendelssohn, Lessing, and Schiller. No successful European revolution could proceed without dealing with these skeletons; and none did.

—David Shavin

This quick sketch is only a beginning suggestion as to the interplay of: Gauss’s “Disquisitiones Arithmeticae”; the healthy benefits of opposing evil (e.g., the imperial Beast-Man, Napoleon); the children and grandchildren both of Moses Mendelssohn and of the American Revolution in Europe; and the passion of magnetic measurements and the revival of Bach’s “St. Matthew Passion.” Much more can, and should be covered in this specific period, regarding the activities of J.F. Cooper, J.Q. Adams, Lafayette, Friedrich List, E.A. Poe, et al. But this abbreviated historic sketch, centered around Dirichlet, should return us, somewhat refreshed, to the Gauss/Dirichlet/Riemann dialogue presented in Part 1 of this Pedagogical Exercise. —DS

1. J.S. Bach had composed and performed this work in Leipzig, in 1729. The manuscript was given to Felix by his aunt Sarah Itzig Levy, a proponent of Bach. Otherwise, one could say that it was fortunate Felix Mendelssohn had exactly 16 friends to cover the four quartets of soprano/alto/tenor/bass, but it were more likely that the orbit defined the planet; that is, that the Bach project cemented the potential friendships.
2. Quotations from Fanny Mendelssohn as reported in François Tillard, *Fanny Mendelssohn*, trans. by Camille Naish

(Portland: Amadeus Press, 1996).
3. Gans was a Jewish student of Hegel. See, Steven P. Meyer, “Moses Mendelssohn and the Bach Tradition,” *Fidelio*, Summer 1999 (Vol. VIII, No. 2).
4. Quoted in Heinrich Eduard Jacob, *Felix Mendelssohn and His Times* (Englewood Cliffs, N.J.: Prentice-Hall, 1963), p. 89.
5. Larchet is unknown to this author. Since it is thought that Dirichlet’s parents were active republicans who had to leave Napoleonic France years before, and since Larchet de Charmont was a friend both of Foy and of Dirichlet’s parents, it were

likely that they were all, indeed, anti-Napoleon republicans.
6. Sebastian Hensel, *The Mendelssohn Family*, trans. by Carl Klingemann (New York: Harper & Brothers, 1881), 2nd rev. edition, Vol. I, p. 312.
7. *Ibid.*
8. Recall that it was Dumas who made the knowing allusion, as part of Dumas’ typically “factitious” fiction, to Poe’s stay in Paris. This is the reference that Allen Salisbury reported on years ago in his “Edgar Allan Poe, The Lost Soul of America,” *The Campaigner*, June 1981 (Vol. 14, No. 3).