extremely important, owing to the widespread, but totally fallacious claim, that Classical music evolved "naturally" toward the atonal cacophony of so-called modern music. In fact, far from being a step *toward* arbitrary chromaticism, the C-F♯-based Lydian mode, as understood by Mozart and Beethoven, achieves an enormous increase in the "Cantorian" ordering-power of tonal composition. Thereby it became possible to eliminate any remnants of arbitrary chromaticism that might otherwise be hiding between the toes of the earlier major-minor system.

1. What is commonly referred to as "melody," including so-called solo melody, is nothing but a derived feature of vocal polyphony. Strictly speaking, *monophonic melody does not exist.* What we call the melody of a solo voice, for example, is nothing but that voice's singing of an intrinsically polyphonic composition. A relevant reflection of J.S. Bach's views on the polyphonic principles of so-called melodic (or better, motivic) development, is contained in the first biography of Bach, written by Nicolaus Forkel ["On Johann Sebastian Bach's Life, Genius, and Works," in *The Bach Reader,* ed. by Hans T. David and Arthur Mendel (New York: W.W. Norton, 1966)]. Otherwise, the cases of Gustav Mahler and Richard Wagner typify the way in which, as soon as composers depart from the rigorous principles of well-tempered polyphony, their melodies degenerate into nothing but ugly groaning.

2. In Book III of his *Harmony of the World,* Kepler polemicized against the empiricist, mechanical theory of musical consonance and dissonance, which had been put forward by Vincenzo Galileo, the father of Galileo Galilei. Vincenzo is regarded as the pioneer of the reductionist musical theory later associated with Jean Le Rond d'Alembert (1718-1783) and Jean-Philippe Rameau (1683-1764), which became virtually hegemonic by the end of the Nineteenth Century, thanks to Hermann Helmholtz (1821-1894).

3. Further exploration of this point might usefully focus on the significance of vibrato in the *bel canto* singing voice—a vibrato which, in strong contrast to the Romantic's pathetic tremolo, is defined as a variation of pitch within a well-tempered pitch-corridor. Apart from the role of vibrato in the technique of *bel canto* singing, one can demonstrate how passages sung without the vibrato, i.e., at a "mathematically fixed" pitch, are correctly heard as *wrong,* destroying the fabric of explicit and implied cross-voice relationships.

3 The Scientific Discoveries of Bach's *The Art of the Fugue*

by Renée Sigerson

Johann Sebastian Bach's *The Art of the Fugue* forces us to become aware of the ontological character of the relationship, in musical composition, between the principle underlying generation of the Lydian mode, and broader applications of the principle of inversion. To most readily appreciate this, it is important to grasp the term "principle" in respect to LaRouche's conception of revolutionary axiomatic progress, whereby the development of man's knowledge of discovered and realized Classical-artistic principles advances, anti-entropically, as expressed by the function *(m+1)/m.*

Usually, musicians only consider inversion as a "technique" of counterpoint, or as an "element" of composition, and not as bearing upon *principles* of discovery. Thus, the import of Bach's work in *The Art of the Fugue* has until now been appreciated only by a few great composers. While there are certain difficulties that need to be overcome to know this composition, it is nonetheless a transparent composition, which excellently illustrates LaRouche's discussion of the generation of new, valid metaphorical principles.

The progress of hypotheses in the composition occurs, in first approximation, as one moves from one fugue to the next in the series, and from one set of fugues to the next. The current discussion focusses on the discovery unveiled in Fugue IV, relative to Fugue I, with some reference to Fugue III.

Preliminarily, it is possible to summarize that discovery as follows: Bach demonstrates, in the "unfinished business" left over from Fugue I and realized in Fugue IV, the generative significance for *all* keys, of the F♯ *major/minor* mode, which is derived from the register shift of the soprano voice. The F♯ major-minor modality is demonstrated as an *extension* of the simple Lydian modality. In other sections of this report, we show that the simple Lydian modality, centered on F♮, arises from inverting the C major scale. In Fugue IV of *The Art of the Fugue*, Bach demonstrates that there is a higher principle involved, in the deceptively simple effort to shift the F♮ Lydian modality to the locus of F♯, the soprano register shift.

As W.A. Mozart clearly grasped (although he reportedly never saw *The Art of the Fugue* manuscript itself), Bach's conception of inversion, exemplified in this extension of the Lydian principle, allowed for a much greater density of lawful change. Bach's use of inversion *across voices,* incorporating the significance of registral transformation and inversion as a unified, single type of principle embedded within the well-tempered system, had a far-reaching impact upon Mozart's own ideas.

The introduction of a manifold of keys around F♯ minor occurs in the critical passage beginning measure 72 of Fugue IV, resolving to C major in measures 86-87 (see below). The discovery and situating of the F♯ mode, is the product of a revolution of axiomatic principles, which begins with the paradoxical implications of a discovery in Fugue I. Any ensemble of musicians attempting to play Fugue IV necessarily experiences the referenced passage as having bearing upon Ludwig van Beethoven's late string quartets.

As we present the musical demonstration of this discovery, it will be useful to keep the following excerpts from Lyndon LaRouche's main essay, "The Substance of Morality," in mind:

"With Plato, one begins with propositions being entertained as prospective theorems, and then follows the approach taken in his dialogues, as a way of searching out discoverable fallacies in those underlying presumptions. . . . The challenging of such prejudices, provides the user of Plato's method with what appears to be, for the moment, a refined array of mutually non-contradictory definitions, axioms, and postulates; this refined array, taken as a whole, is an *hypothesis*. . . .

"The method of Plato starts with the recognition that all . . . hypotheses, including what were previously the most refined ones, must include some significant, axiomatic fallacy of some kind. . . .

"Truth, then, does not lie in any one choice of hypothesis. . . . Truth lies in the always radically revolutionary process, by means of which valid new

Comments on Bach's Fugues by His Contemporaries

'He who is not acquainted with Bach's fugues cannot even form an idea of what a true fugue is and ought to be. In fugues of the ordinary kind, there is nothing but a certain very insignificant and sloppy routine [*Schlendrian*]. They take a theme, give it a companion, transpose both gradually into the keys related to the original one, and make the other parts accompany them in all these transpositions with a kind of thorough-bass chords. This is a fugue; but of what kind? . . . Bach's fugue is of quite another kind." (Johann Nicolaus Forkel, "Biography of J.S. Bach," in *The Bach Reader,* ed. by Hans T. David and Arthur Mendel [New York: W.W. Norton, 1966], p. 324.)

'In composition, [Bach] started his pupils right in with what was practical, and omitted all the *dry species* of counterpoint that are given in Fux and others. His pupils had to begin their studies by learning pure four-part thorough-bass. From this he went to chorales; first he added the basses to them himself, and they had to invent the alto and tenor. Then he taught them to devise the basses themselves. He particularly insisted on the writing out of the thorough-bass in parts. In teaching fugues, he began with two part ones and so on." (Letter from Carl Philipp Emanuel Bach to Forkel, in *The Bach Reader, op. cit.,* p. 279.)

'The true fugue is two sorts, distinguished according to their treatment of the fugue subject:

"(A) A strict fugue, *fuga obligata,* is one in which no other material than the subject is treated throughout, i.e., in which the subject after the exposition . . . makes its appearance in one entry after another, so to speak, and in which, consequently, all the counterpoints and interludes are derived from the principal subject or from the counterpoint that first appears against the answer, by means of division, augmentation, diminution, contrary motion, etc.; all this however, being bound together through imitation and a coherent and solid harmony. When such a strict fugue is worked out at length, and all kinds of other artifices (made possible by the many kinds of imitation, double counterpoint, canon, and change of key) are introduced in it, such a piece is called by the Italian name of *Ricercare* or *Ricercata*—an art fugue, a masterfugue. Such is the nature of most of the fugues by the late Capellmeister Bach.

"(B) A free fugue, *fuga libera, solu-ta, sciolta,* is a fugue in which the principal subject is not continuously treated; that is, in which it does not make its appearance in one entry after another, although often enough, and in which, when the principal subject is abandoned, a brief, well-chosen interlude is worked out by imitation and transposition—which has a similarity to the principal subject or to the counterpoint that first appears against the answer, and is related to the same, even though it is not always derived from it. Such is the nature of most of the fugues by Handel." (Friedrich Marpurg, 1753, quoted in *The Bach Reader, op. cit.,* p. 254.)

[**Note:** Marpurg was no friend of Bach's. While his distinction between free and strict fugue is somewhat useful, he, a typical musicologist, thinks in terms of form, not ideas. In fact, *A Musical Offering* is of the character he indicates, but the fugues of *The Art of the Fugue* are much more groundbreaking and complex. The useful distinction to be made, is between Bach's type of thinking, and the sort of fugues Haydn wrote, before 1782. Examine, for example, Haydn's String Quartet in F minor Op. 20, No. 5: Every entrance is on a Lydian interval, but the principle associated with the Lydian mode is not even referenced.]

FIGURE 3.1

Opening of Fugue I from J.S. Bach's *The Art of the Fugue*

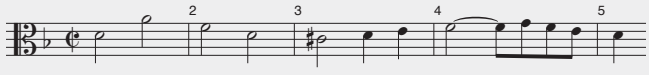


FIGURE 3.2

Counter-subject of Bach's final fugue

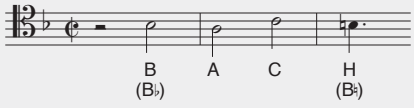
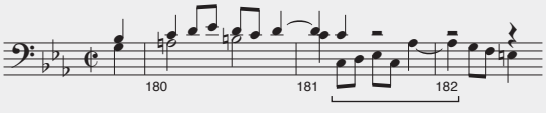


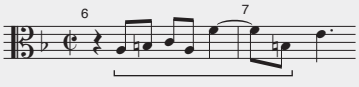
FIGURE 3.3

From *A Musical Offering* to *The Art of the Fugue*

From *A Musical Offering,* six-part Ricercar:



From *The Art of the Fugue,* Fugue I:



principles are generated, new principles which take into account the contradictions inhering in the previously proposed hypothesis."

Later in the same essay, LaRouche writes:

"We have to consider the cases, in which a particular colligating set of principles is in error only because it lacks some additional principle."

Relationship to *A Musical Offering*

According to all accounts, *The Art of the Fugue* was composed by J.S. Bach in the year of his death. It consists of 19 fugues. It was written less than two years after his *A Musical Offering*. *A Musical Offering* centers around a *six-voice* fugue; *The Art of the Fugue* contains fugues with four, three, or two voices. Many musicians have complained about the form in which the manuscript was written, in "open score," with each part on a separate line, and its own unique clef. This makes it difficult to play the composition, at sight, on keyboard. However, it was necessary for Bach to leave the primary manuscript in that form. It forces the musician working with the composition to always think of the individual part as associated with a voice species, such as soprano, alto, tenor, or bass. Usually, which voice is intended, is adequately indicated by the clef and line on which the voice appears. That, however, is not *always* the case, so that sometimes the top voice may in fact be indicating an alto part, for example. The reason is that in the actual working through of the composition, sometimes Bach is working, implicitly, with the full *six* voices of the *Musical Offering* in mind.

The unifying thread of development across the 19 fugues, is encompassed in the seeming irreconcilability of the opening, and the concluding ideas of the entire work. That opening idea is the deceptively simple, seemingly inconsequential four-measure phrase that opens Fugue I **(Figure 3.1)**.

By Fugue XIX **(Figure 3.2**), Bach has arrived at what was undoubtedly his originally intended goal, a fugue containing a countersubject spelling out his own family name.

When thinking of the great distance one will traverse, from the opening statement, to that conclusion, there is to be remembered the awe-inspiring inscription engraved by Bach's son, Carl Philipp Emanuel, onto the original manuscript plate: "N.B. While working on this fugue, where the name BACH appears in the countersubject, the composer died." (In German, the letter "H" denotes B♮, and "B" denotes B♭.) Thus, when we think of *The Art of the Fugue*, we are virtually compelled to remember that Bach immersed himself in this composition, in his final moments, giving to future generations the benefit of his knowledge. In that respect, Bach exemplifies LaRouche's view of a world historical personality.

Now, we enter upon the demonstration.

In all of the opening four fugues, the opening four-measure statement is always followed by a fragment, which is essential to the unfolding of a manifold of ideas. In the case of Fugue I, that fragment is, with one change, lifted directly out of the 1748 *Musical Offering* composition **(Figure 3.3)**. Note the difference introduced in the later fragment. In the *Art of the Fugue*, the f♮′ at the top of the phrase descends by a *Lydian interval* downwards to b♮. Thus, the material has been transformed to incorporate the principal discovery of the *Musical Offering*, the Lydian principle, as a point of reference.

In Fugue I, as each voice enters, an inherent paradox emerges, showing that the original idea was not as "inconsequential" as may have appeared. The half-step motion into the third measure of the theme (e.g., in the first statement, d′ down to c♯′) generates a sequence of paradoxical cross voices, dominated by the Lydian interval. The purpose of the original "fragment" revised from the *Musical Offering* becomes clear: to prepare the mind for the sequence of Lydian intervals that will occur—for example, in the passage shown in **Figure**

3.4a—or, when all four voices have finally entered, on measure 15, the voices form a double Lydian interval of f-g♯-d′-b′ **(Figure 3.4b)**.

The tension between the original idea, rooted in D minor, and the Lydian intervals, which imply motion toward any number of potential modes, requires the introduction of a new idea, to forge progress. The idea introduced by Bach is a rising fourth, which begins to predominate and shape the direction of the earlier material. The rising fourth becomes pervasive throughout the entire fugue.

The passage beginning measure 36 **(Figure 3.5)** exemplifies this approach, in the way the bass voice is organized. Do not think that this is somehow the first time the fourth appears in the score, for it is not. That is not the point. Rather, the emerging predominance of the fourth occurs in the same way that, in a drama, a character in the background—perhaps a member of a crowd—suddenly steps forward and plays an important role. Bach's determination that a *third* idea must always be introduced in these fugues, underlines the difference between his concept of fugue, as well as of music overall, relative to lesser composers.

Prior to Bach, many other composers "used" the Lydian interval, but only as "another" device, or "element" of composition. The underlying principled importance of the Lydian as discussed in other sections of this report, eluded them. Unlike Bach, they confined themselves to writing "strict" fugues, where the theme would be repeated, then inverted, or changed rhythmically; but there was no ordering principle governing the *ideas* of the composition.

Let us take another example, in which the role of the fourth becomes even more significant. The passage in measures 36-40 **(Figure 3.5)** concludes with a very strong resolving interval: in which e in the bass voice moves *downward* by a *fifth* to A. This is very important, for the following reason: The *inversion* of that interval, e moving upward a *fourth* to a, is the high-point of the movement. In measure 49

FIGURE 3.4
Lydian intervals introduced in Fugue I

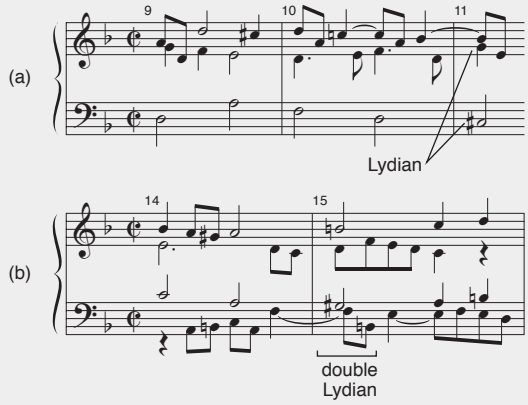


FIGURE 3.5
Fugue I, measures 36-40

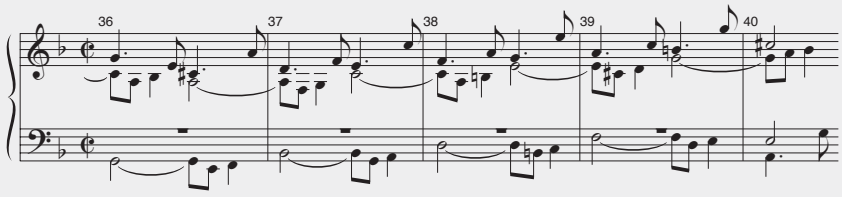


FIGURE 3.6
Fugue I, theme introduced in highest voice

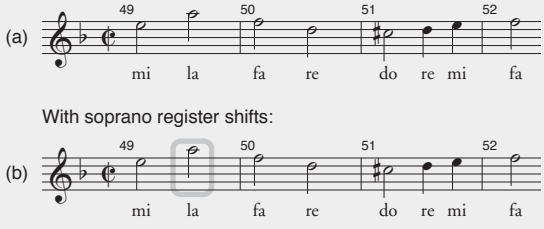


FIGURE 3.7
Conclusion of Fugue I

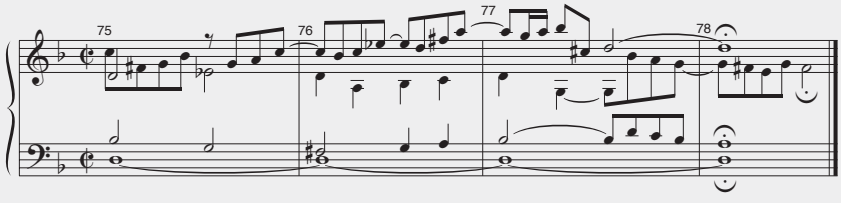


FIGURE 3.8

Art of the Fugue **theme and elementary inversion**

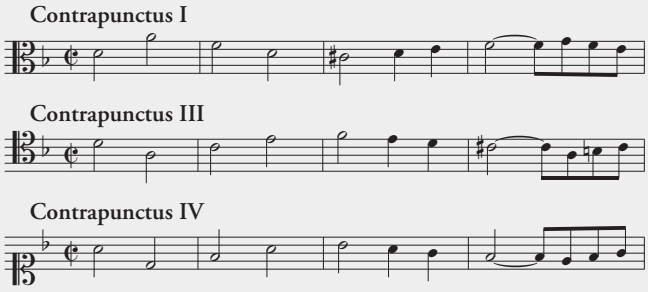
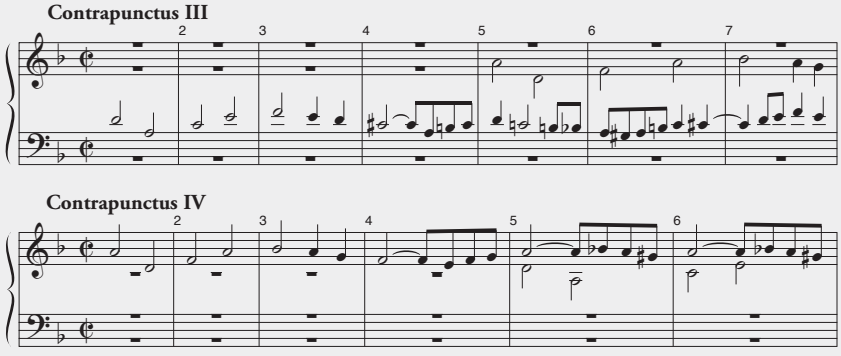


FIGURE 3.9

Fugues and fragment counter-subjects, III and IV



(Figure 3.6a), the topmost voice introduces the theme, though this time introduced by the interval of the fourth, situated as e″ moving upward to a″. Thus, the soprano *inverts* the earlier bass voice resolution, referenced above (the last beat of measure 39 going into measure 40). This soprano *inversion* of the bass voice resolution is a turning point in the movement. These are the "highest" tones on which the soprano voice introduces the theme. Even more significantly, in this position, the uppermost voice is presenting the theme, for the first time, in such a way as to *cross* from the second to third register of the human soprano singing voice **(Figure 3.6b)**.

This coupling of inversion with registral differentiation—including across two different voice species, namely bass to soprano—is essential to what is meant by *ontological principle* in musical composition. Although these fugues are "instrumental" works, the underlying conception is entirely consistent with the *bel canto*-trained "chest" of human voices. Vocal registration is an ontological characteristic of musical art (see Chapter 1). As we shall see, Bach was intently focussed on the implications of the *difference* implied between a particular interval, that interval in respect to its inversion; and that pair of inverted intervals relative to changes of vocal registration, in different voice parts, as reflecting *ontological principles* of musical composition.

The importance Bach attributes to the shift in soprano vocal registration, is indicated by the final four measures of Fugue I **(Figure 3.7)**, in which the soprano voice evokes a *cadenza* passage. Though there is a d pedal-point in the bass, the soprano voice is spelling out an ascending C minor "scale." This "scale" is actually composed of the identical material upon which the *Musical Offering* is based, namely, the paired Lydian intervals of C-F♯ and E♭-A. (Note, furthermore, the downward diminished seventh from the high b♭″ to the c♯‴ at the end of the phrase, again an interval readily identified with the *Musical Offering*.)

Now, to have a clearer view of the principle indicated—and to experience its profound implications—we turn to Fugue IV.

First, a chart which simply situates the reader in respect to the material **(Figure 3.8)**. In Fugue I, the theme is ascending. In Fugues III and IV, the theme is *inverted* to assume its *descending* form. The *inversions* denote the onset of more developed hypotheses, inclusive of the emergence of new constructive principles of composition. Bach's recognition that *inversion* required such a development of new hypotheses, is what distinguishes his concept of fugue, from schoolbook versions of "strict fugue."

In private discussion, Lyndon LaRouche has pointed out that *The Art of the Fugue* properly situates what is often called "chromatic" motion. Throughout the composition, Bach shows that "chromatic" motion is not some kind of sensual effect, but rather is a necessary *theorem of inversion.* This is particularly evident in the canonical duet, Fugue XV, not shown here.

Important to our investigation, is that both Fugues III and IV, which are inversions of the opening idea, introduce as companions to the root theme, *chromatic* countersubjects, that is, phrases based on motion by half-step.

To make this clear, we show again the opening measures of Fugues III and IV **(Figure 3.9)**, accompanied by their *fragment* countersubjects, which are quite different from the fragment discussed in respect to Fugue I. Consider for one moment the "chromatic" fragment attached to Fugue IV: Implicitly this is a statement of inversion. The middle tone is a root. The half-step above and below the middle tone are moving in inverted directions from one another (i.e., the g♯ at the end of measure 5 moves back *up* to the a).

Immediately, the propositions being presented in Fugue IV are more densely organized, per interval of action, than those in Fugue I. That should not surprise us, since what Bach is pursuing here, is to further develop the "unfinished" question left over from Fugue I.

This greater density of principles is exemplified by the soprano voice in measure 13 **(Figure 3.10)**. At this early point in the composition, the soprano voice moves into the third register, directly referencing the poetic high-point of Fugue I. The reference to the Fugue I is explicit. The soprano moves exactly as before, upwards by a fourth, from e″ to the third-register a″.

From this point on, there is a much greater density of interaction between colligating principles, relative to Fugue I. The reason for that will become clear.

For example: As in Fugue I, Bach will introduce a "new" interval, to re-situate the paradox created by the fugue theme placed against its countersubject (in this case, the step-wise [chromatic] motion). Here, the "new" interval is not a rising fourth, but rather a *descending* third, consistent with the fact that throughout Fugue IV, the overall direction of everything (except the soprano voice!) is *downward*.

Note, however, that this descending third is an interval of a more complex type than the fourth in Fugue I. Why? Because Bach always presents the third in duplicate, across two voice parts. For example, in measures 19 through 23 **(Figure 3.11)**, the soprano and alto voices are in such a dialogue. Implicitly, the paired dialogue of descending thirds is spelling out an inverted fifth, or, sometimes, Lydian interval. Thus, implicitly, the paired thirds occur as an inversion of the fourths and fifths up to this time.

There is an additional clue concerning the purpose of this process. In both fugues, there is a significant occurrence of Lydian intervals. In this fugue, however, Bach meticulously *postpones* the introduction of the interval F♯-C, until well into the development of the composition. Despite one early reference to C-G♭, the interval F♯-C *only* occurs in respect to the soprano voice entering the third register!

For example, the passage beginning the second half of measure 34, through 37 **(Figure 3.12)**, illustrates this pairing of the Lydian interval c″-f♯′ with the evoking of the soprano third register. In the course of measure 37, for the first time, the double Lydian sequence a′-e♭′-c″-f♯′ occurs, explicitly spelled out in the bass and alto voices. The soprano voice, meanwhile, is entirely in the third register!

This occurs yet again, in measure 63-64 **(Figure 3.13)**. The soprano is crossing back and forth between the second and third registers. As it does so, the *tenor* voice executes a remarkable Lydian interval: e♭′ down to a (obviously closely related to the ascending soprano e♮″ up to a″). As it enters on the a, the same double Lydian sequence occurs across the four voices: f♯-a-c′, with the e♭′ from the tenor's

FIGURE 3.10
Soprano and alto voices, Fugue IV, measures 13-14

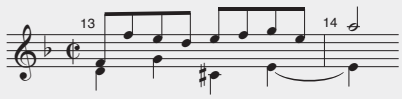


FIGURE 3.11
Pairs of descending thirds in Fugue IV

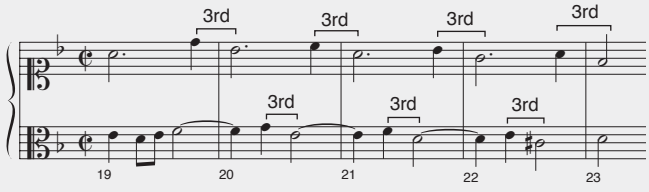


FIGURE 3.12
Fugue IV, measures 34-37

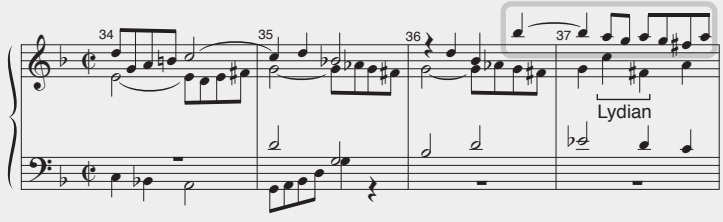


FIGURE 3.13
Fugue IV, measures 63-64

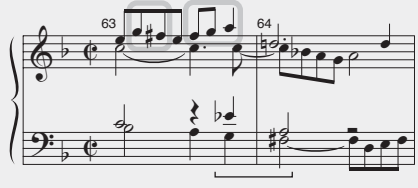
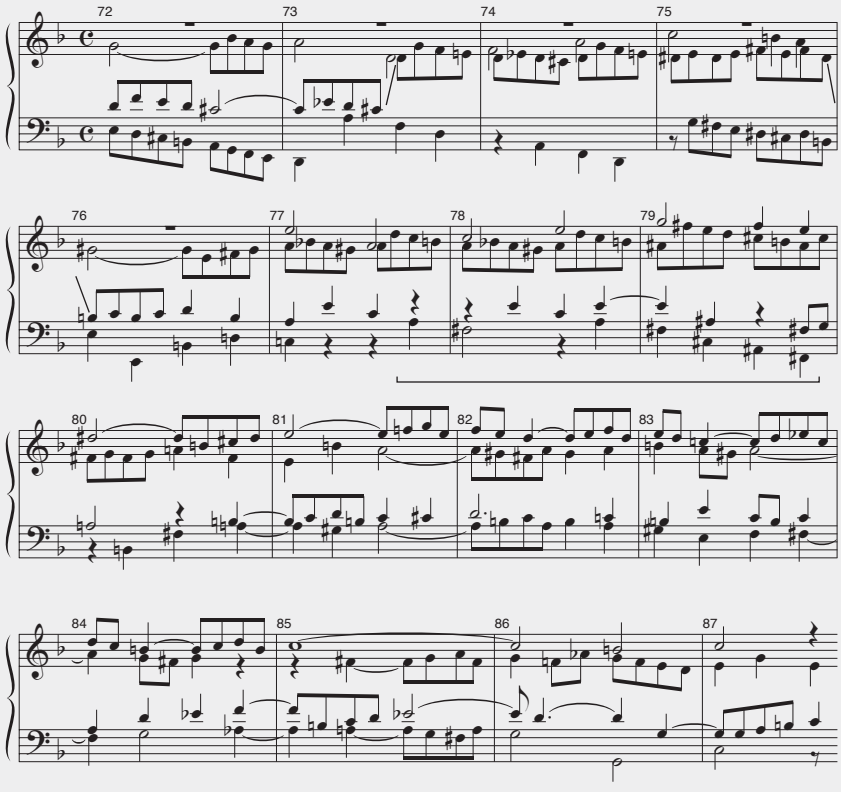


FIGURE 3.14

Fugue IV, measures 72-87



previous tone implicitly included in the paired Lydian intervals.

Now, we have arrived at the "*punctum saliens*" of this fugue. Something fairly remarkable is about to occur.

According to different source materials, Ludwig van Beethoven copied several measures from this section of Fugue IV into a notebook associated with his late string quartets. His entry includes measure 61, then a double slash on the staff to indicate a jump, and then three and a half measures beginning with the second half of measure 72. (See Figure 7.2 for a transcription of these passages from Beethoven's notebook.)

The passage beginning with measure 72, through to the C-major resolution in measures 86-87 **(Figure 3.14)**, records a demonstration of the process whereby a new principle is introduced to the manifold of validated metaphorical discoveries [*(m+1)/m*], not merely as such discoveries have bearing within a particular musical composition, but, rather those higher order discoveries which bear upon the entire domain of musical art.

What Bach shows, is that the "solution" to the unresolved crossroad of the relation of the soprano register shift to the set of contrapuntal problems posed thus far, lies outside the domain of what might be called "contrapuntal" considerations. Beginning with the measure copied by Beethoven, the bass voice descends to the lowest pitch for its voice in this composition, a low D. Against the backdrop of two references to the interval C-F♯, for the next seven measures, the composition becomes, in stages, increasingly "blurry." It is almost as if each of the voices "has a mind of its own," typified by the soprano in measure 75 sounding c″, against the c♯ in the bass.

For several measures, there is a keyless mode, until the soprano enters, asserting the opening theme, in the mode of A minor. However, the outcome of this placement of the theme in the soprano voice, is the exact opposite of what one would expect. Rather than the composition becoming more simply ordered, the opposite occurs.

As the soprano moves upward, to a third-register g″, coming down to a second-register f♮″, *all of the other voices are emphasizing F♯*, the precise value at which the third register shift occurs. The blurrying motion intensifies, and a heretofore unknown mode, F♯ major/minor, against B minor, takes over. After five measures of this treatment, everything comes together around C major!

This passage evokes precisely the "eerie" quality LaRouche discusses in respect to great tragedy—where one becomes conscious that it is the ideas hovering above the individual elements of composition which are governing the development. In the case of music, at this precise moment, one cannot help but think of Beethoven's late quartets, even if one knew nothing of the passing reference to this fugue passage found in Beethoven's sketchbook.

What is Bach proving to us? Do not look for the root of great musical composition in the formalities of counterpoint, or in any *one* of the principles. Rather, seek the root of composition in the generative capacity to improve the principles which bring these elements together. It is actually the *irony* of the soprano register shift, emphasized here by the interplay across the passage of F♮ against F♯, relative to other colligating principles, which is driving Bach to focus on the underlying method of ordering these principles. Density is demonstrated by the very compact way Bach moves from the "eerie" realm of F♯ major/minor to the resolution of C major. In so doing, he has extended the notion of Lydian principle, in a most profound, and scientifically valid way, by emphasizing its ontological root in the soprano register shift. This also has extended his conception of inversion.